Air temperature equation derived from sonic temperature and water vapor mixing ratio for the air flow through closed-path eddy-covariance systems





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At high frequency, a closed-path eddy-covariance system measures sonic temperature in open space and water mixing ratio in the closed measurement camber with more stable temperature and pressure than in an open measurement space.

54 Street



Air temperature derived from sonic temperature and water mixing ratio in a closed-path eddy-covariance system has advantages over the temperature probe measured one in reflecting its high frequency fluctuations needed in flux calculations and in minimizing its solar contamination for measurement quality.

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 $=\frac{I_s}{\left(1+0.51q\right)}$ 

- T air temperature
- $T_s$  sonic temperature
- *q* specific humidity
- e water vapor pressure
- *P* atmospheric pressure

Schotanus et al. (1983)

 $\frac{1}{1+0.32\frac{e}{r}}$ 

Kaimal and Gaynor (1993)



#### **SND** equation

$$T = \frac{T_s}{\left(1 + 0.51 \frac{\rho_w}{\rho_d + \rho_w}\right)} = T_s \left(1 + 0.51 \frac{\chi_w}{1 + \chi_w}\right)^{-1}$$

- T air temperature
- $T_s$  sonic temperature
- $\rho_d$  dry air density
- $\rho_w$  water vapor pressure
- $\chi_w$  water vapor mass mixing ratio

#### **KG** equation

$$T = \frac{T_s}{\left(1 + 0.32 \frac{R_v T \rho_w}{R_d T \rho_d + R_v T \rho_w}\right)} = T_s \left(1 + 0.51 \frac{\chi_w}{1 + 1.61 \chi_w}\right)^{-1}$$



- *T* air temperature
- $T_s$  sonic temperature
- $\chi_w$  atmospheric pressure

 $\Delta T$  = |SND equation – KG equation|

$$\Delta T = \frac{0.31T_s \chi_w^2}{1 + 3.63\chi_w + 3.20\chi_w^2}$$

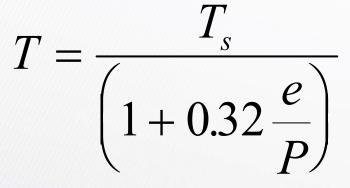
$$T_{s}$$
: -30 ~ 57 °C  
 $\chi_{w}$ : 0~ 45 g kg<sup>-1</sup>  
 $\Delta T$ : 0 ~ 0.176 °C



$$T = \frac{T_s}{\left(1 + 0.51q\right)}$$

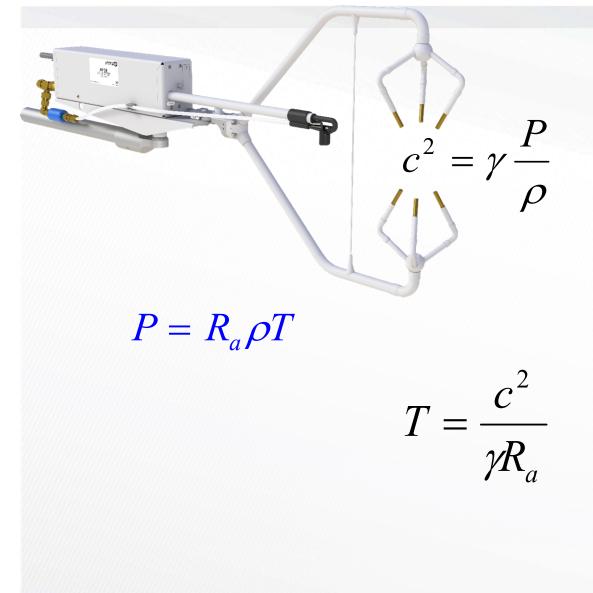
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- $T_s$  sonic temperature
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Schotanus et al. 1983



Ishii (1932) Barrent and Suomi (1949) Kaimal and Gaynor (1993)





#### Barrrett and Suomi (1949)

- c speed of sound
- γ ratio of moist air specific heat between constant pressure and constant volume
- *P* atmospheric pressure
- $\rho$  moist air density
- $R_a$  gas constant of moist air
- *T* air temperature



 $T = \frac{c^2}{\gamma R_a}$ 

 $T_s = \frac{c^2}{\gamma_d R_d}$ 

- *T* air temperature
- $T_{\rm s}$  sonic temperature
- c speed of sound
- γ ratio of moist air specific heat between constant pressure and constant volume
- $\gamma_d$  ratio of dry air specific heat between constant pressure and constant volume (1.400279)
- $R_a$  gas constant of moist air
- $R_d$  gas constant of dry air (287 J K<sup>-1</sup> kg<sup>-1</sup>)

Sonic temperature of moist air is the temperature that its dry air component can reach at the same enthalpy as the moist air has.



## Equation derived from the first principals without any approximation

$$T = T_{s} \frac{\left(1 + \varepsilon \chi_{H2O}\right) \left(1 + \varepsilon \gamma_{v} \chi_{H2O}\right)}{\left(1 + \chi_{H2O}\right) \left(1 + \varepsilon \gamma_{p} \chi_{H2O}\right)}$$

- *T* air temperature
- $T_s$  sonic temperature
- $\gamma_v$  ratio of specific heat at constant volume between water vapor and dry air (2.04045)
- $\gamma_p$  ratio of specific heat at constant pressure between water vapor and dry air (1.94422)
- ε ratio of molecular mass between water vapor and dry air (0.622).  $\chi_{H2O}$  water molar mixing ratio

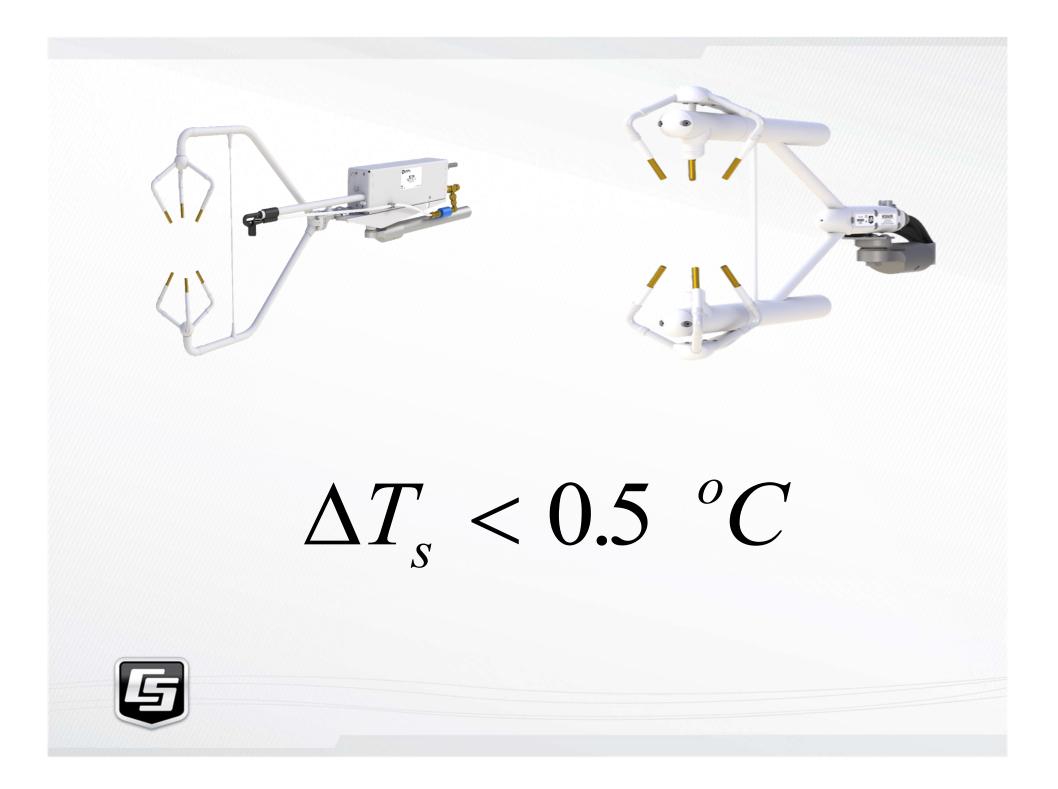


### Error in calculated air temperature

$$\Delta T = \frac{T(T_s, \chi_{H2O})}{T_s} \Delta T_s + T(T_s, \chi_{H2O}) \left[ \frac{\varepsilon + \varepsilon \gamma_v (1 + 2\varepsilon \chi_{H2O})}{(1 + \varepsilon \chi_{H2O}) (1 + \varepsilon \gamma_v \chi_{H2O})} - \frac{1 + \varepsilon \gamma_p (1 + 2\chi_{H2O})}{(1 + \chi_{H2O}) (1 + \varepsilon \gamma_p \chi_{H2O})} \right] \Delta \chi_{H2O}$$

- T air temperature
- $T_s$  sonic temperature
- $\gamma_v$  ratio of specific heat at constant volume between water vapor and dry air (2.04045)
- $\gamma_p$  ratio of specific heat at constant pressure between water vapor and dry air (1.94422)
- ε ratio of molecular mass between water vapor and dry air (0.622).  $\chi_{H2O}$  water molar mixing ratio







### $\Delta \chi_{H2O} = \Delta \chi_{H2O}^{p} + \Delta \chi_{H2O}^{s} + \Delta \chi_{H2O}^{g} + \Delta \chi_{H2O}^{z}$

# precision sensitivity gain drift zero drift to $CO_2$



#### Maximum error ranges of calculated air temperature at different relative humidity

